Jakarta International School

8th Grade – AG1

Practice Test - Black

Linear Systems

Name: Solutions

Date:

Score: $\frac{\quad}{25}$

Goal 5: Solve and apply linear systems

Directions: Use systems of linear equations/inequalities to solve the following problems.

1. A rectangular board is to be constructed to the following specifications.

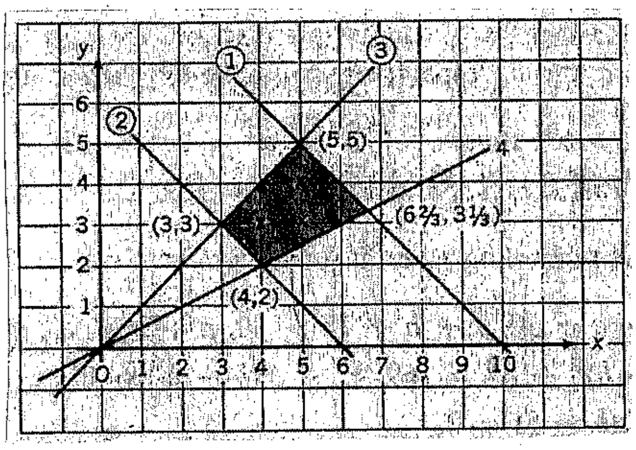
   - the perimeter is equal to or greater than 12 inches, but less than 20 inches
   - the ratio of adjacent side lengths is greater than 1 but less than 2.

   Find all sets of integral (integer values) dimensions satisfying the above specifications (5 points)

Let x = the larger dimension

y = the smaller dimension

$$\left.\begin{array}{ll} (1) & 2x + 2y < 20 \\ (2) & 2x + 2y \geq 12 \\ (3) & \dfrac{x}{y} > 1 \\ (4) & \dfrac{x}{y} < 2 \end{array}\right\}$$



The shaded quadrilateral represents the solutions that are common to all 4 inequalities.

Each x represents a pair of possible integral dimensions.

The possibilities are (4,3) (5,3) and (5,4).

2. **The Farmer in the Dilemma.** Once upon a time in the land of giants and beanstalks, there lived a farmer and his wife. One day the honorable tax collector came to the village and told the people that he was returning \$100 of the tax moneys to them on the condition that it be used to purchase exactly 100 farm animals. The farmer was chosen by the villagers to go into town and buy some cattle, some sheep, and some horses. He was to spend the entire \$100 on animals - no popcorn, jelly beans, or anything else. When he arrived in town, he discovered that sheep cost \$.50 each, cattle cost \$1.00 each, and horses cost \$10 each. Your job is to provide a solution to the farmer's dilemma. (5 points)

There are 5 solutions; one for each of one, two, three, four, or five horses. Let C, S, and H represent the number of cattle, sheep, and horses, respectively.

(1) $C + S + H = 100$ animals

(2) $.5S + C + 10H \overset{\$}{=} 100$

solve (1) for C: $C = 100 - S - H$

substitute C into (2): $.5S + (100 - S - H) + 10H = 100$

simplify $\frac{-.5S}{-.5} = \frac{-9H}{-.5}$

$S = 18H$

Substitute S into (1): $C + 19H = 100^{\bigstar}$

from this equation, we know $H \leq 5$.

When $H = 5$, use $\bigstar$ to get $C = 5$ and then (1) to get $S = 90$.

When $H = 4$, $C = 24$, and $S = 72$.

When $H = 3$, $C = 43$, and $S = 54$.

When $H = 2$, $C = 62$, and $S = 36$

When $H = 1$, $C = 81$, and $S = 18$

The Farmer can choose to buy ~~and~~ any of the above animal combinations.

3. **The Great Doughnut Caper:** Dan's Donut Shop makes the best donuts in town - and makes it easy to get them: Dan accepts orders by phone, even offering home delivery.

One day, Mr. Allen, Mr. Brown, and Mr. Carter each ordered a different number of Dan's donuts; together they ordered five dozen. Dan prepared three boxes. In order to avoid any confusion, as he was filling the boxes he labeled each one with the respective customer's name. When he had filled the orders, Dan stepped back into his kitchen.

While he was out, who should enter the shop but Little Lester, the town prankster. Lester, a 10-year-old scamp, played practical jokes every chance he could.

Lester took 6 donuts out of Allen's box and put them into Brown's box. Before he could do anything to Carter's box, he heard Dan returning, so Lester ducked back out, escaping without Dan being aware of the mischief he had stirred up.

As soon as he learned of the switcheroo, Dan promptly made amends to Mr. Allen. Your task is to determine how many donuts were in each box before Lester entered the scene. After the switch was made, Allen's box contained one-third as many doughnuts as Carter's box; and Carter's box contained one-half as many doughnuts as Brown's box. (5 points)

Let B = Mr. Brown's donuts after Lester made the switch

$\frac{1}{2}B$ = Mr. Carter's donuts which were not tampered with

$\frac{1}{3}\cdot\frac{1}{2}B$ = Mr. Allen's donuts after Lester made the switch.

TOTAL DONUTS = 60, so all the donuts after the switch added to 60.

$$B + \frac{1}{3}\cdot\frac{1}{2}B + \frac{1}{2}B = 60$$

$$\frac{5}{3}B = 60$$

$B = 36$ So after the switch, Mr. Brown's box had 36 donuts, Mr. Carter had 18, Mr. Allen had 6.

Lester had taken 6 donuts out of Mr. Allen's box and put them in Mr. Brown's box and he made no changes to Mr. Carter's box.

$36 - 6 =$ 30 donuts originally in Mr. Brown's box

$6 + 6 =$ 12 donuts originally in Mr. Allen's box.

18 donuts originally in Mr. Carter's box.

Complete the following Linear Programming problems on separate pieces of graph paper

For each problem, complete the following steps:

1. Define your unknowns
2. Express the objective function and the constraints
3. Graph the constraints
4. Find the corner points to the region of possible solutions
5. Evaluate the objective function at all the feasible corner points

4. A gold processor has two sources of gold ore, source A and source B. In order to keep his plant running, at least three tons of ore must be processed each day. Ore from source A costs $20 per ton to process, and ore from source B costs $10 per ton to process. Costs must be kept to less than $80 per day. Moreover, Federal Regulations require that the amount of ore from source B cannot exceed twice the amount of ore from source A. If ore from source A yields 2 oz. of gold per ton, and ore from source B yields 3 oz. of gold per ton, how many tons of ore from both sources must be processed each day to maximize the amount of gold extracted subject to the above constraints? (5 points)

See attached

5. A publisher has orders for 600 copies of a certain text from San Francisco and 400 copies from Sacramento. The company has 700 copies in a warehouse in Novato and 800 copies in a warehouse in Lodi. It costs $5 to ship a text from Novato to San Francisco, but it costs $10 to ship it to Sacramento. It costs $15 to ship a text from Lodi to San Francisco, but it costs $4 to ship it from Lodi to Sacramento. How many copies should the company ship from each warehouse to San Francisco and Sacramento to fill the order at the least cost? (5 points)

See attached.

#4) 1. let x = the number of tons from source A
y = the number of tons from source B

2. The objective is to maximize the amount of gold yield (the gold produced). Since each ton of ore from source A yields 2 oz. of gold and each ton of ore from source B yields 3 oz of gold, the amount of gold recovered will be

$$2x + 3y = \text{Gold Yield}$$

<u>Constraints</u>

Processing $x + y \geq 3$

Cost $20x + 10y \leq 80$

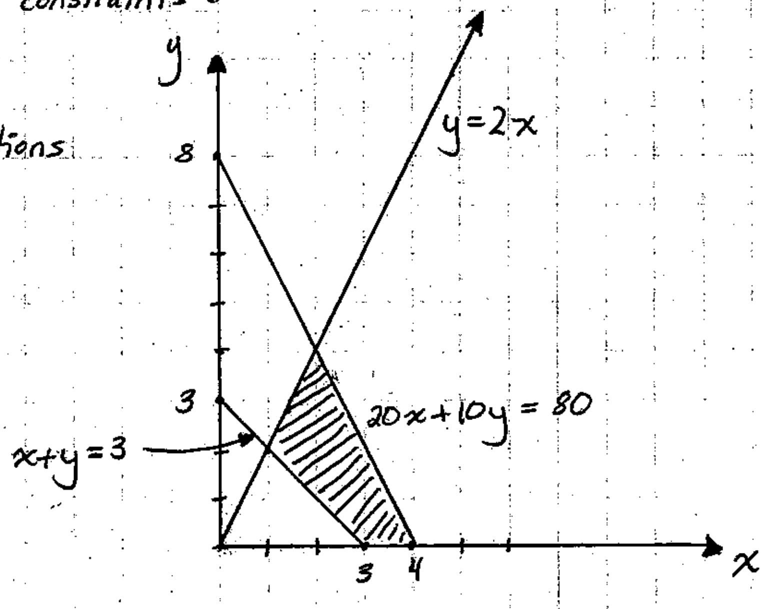
Federal Regulations $y \leq 2x$

A negative number of tons of ore will not be processed

$x \geq 0$ and $y \geq 0$

3. Graph the constraints:

Shade the common solutions



4. Find the feasible corner points: (3,0) (4,0) (1,2) and (2,4)

5. Evaluate the objective function at each corner point:

$(1,2) \rightsquigarrow 2(1) + 3(2) = 8$ oz of gold
$(2,4) \rightsquigarrow 2(2) + 3(4) = 16$ oz of gold ★
$(3,0) \rightsquigarrow 2(3) + 3(0) = 6$ oz of gold
$(4,0) \rightsquigarrow 2(4) + 3(0) = 8$ oz of gold

The maximum yield of gold is 16 oz. by processing 2 tons of ore from source A and 4 tons from source B.

#5 1. let x = the number of books from Novato to San Francisco
y = the number of books from Novato to Sacramento
z = the number of books from Lodi to San Francisco
w = the number of books from Lodi to Sacramento

2. The objective is to minimize the cost

$$cost = 5x + 10y + 15z + 4w$$

<u>Constraints</u>: <u>Orders</u> from San Francisco $x + z = 600$ <u>OR</u> $z = 600 - x$ (1)

<u>Orders</u> from Sacramento $y + w = 400$ <u>OR</u> $w = 400 - y$ (2)

<u>Supplies</u>
There are only 700 books in Novato $x + y \le 700$ (3)
There are only 800 books in Lodi $z + w \le 800$ (4)

Substitute z and w from (1) and (2) into (4) in order to express (4) in terms of x and y

$$z + w \le 800 \quad (4)$$
$$600 - x + 400 - y \le 800$$
$$1000 - x - y \le 800$$
$$200 \le x + y$$

The total order is for 1000 books. There are only 800 books in Lodi. At least 200 books will have to come from Novato.

<u>Implied Constraints</u>: The number of books can never be negative

$$x \ge 0$$
$$y \ge 0$$
$$z \ge 0$$
$$w \ge 0$$

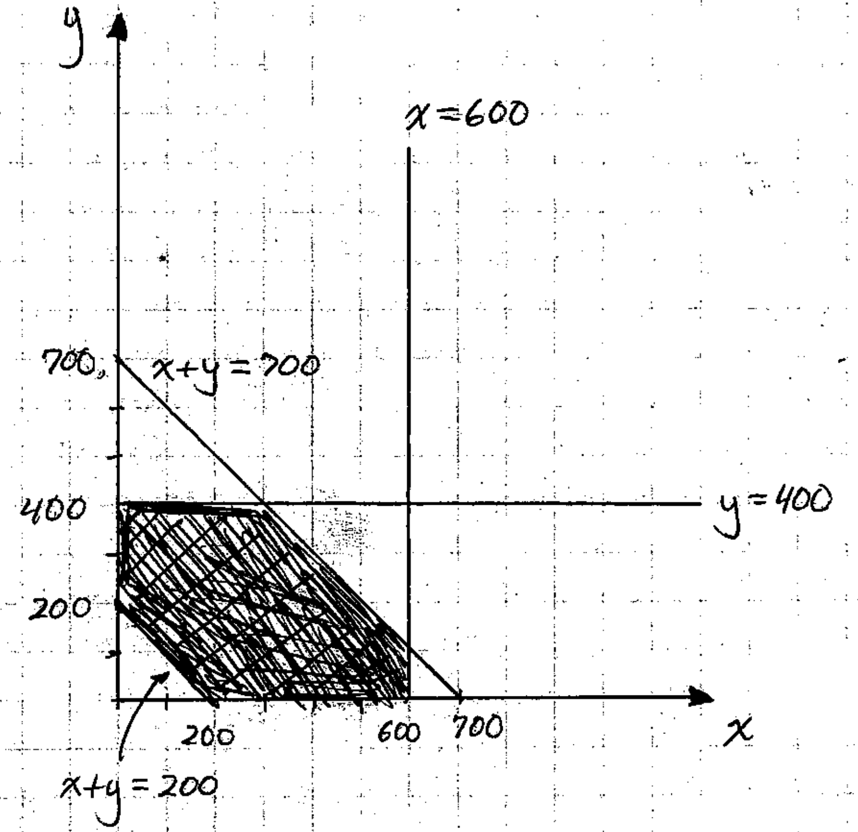
Substitute for z and w: $600 - x \ge 0$, $400 - y \ge 0$

<u>OR</u> $600 \ge x$
$400 \ge y$

So, here is a list of all constraints expressed in terms of x and y.

$$x + y \le 700$$
$$x + y \ge 200$$
$$x \le 600$$
$$y \le 400$$
$$x \ge 0$$
$$y \ge 0$$

3. Graph the constraints



4. Feasible Corner Points:

(0, 400)
(0, 200)
(200, 0)
(600, 0)
(600, 100)
(300, 400)

5. Evaluate the objective function

cost $= 5x + 10y + 15z + 4w$

Remember $z = 600 - x$
$w = 400 - y$

(0, 400) → \$13,000
(0, 200) → \$11,800
(200, 0) → \$8,600
minimum — ☆ (600, 0) → \$4600
(600, 100) → \$5200
(300, 400) → \$10,000

The least cost is \$4600 at (600, 0). You are filling the entire San Francisco order with copies from Novato, which is cheaper and filling the entire Sacramento order with copies from Lodi, which is also cheaper.