

Exponents, Radicals, and the  
Pythagorean Theorem

**Goal 8: Apply Exponents, Radicals, and the Pythagorean Theorem**

**\*\* 2 Points Per Problem Unless Stated Otherwise \*\***

1. Solve for x

<p>A. <math>3^x = 9^2 \cdot 3 \cdot 27^3</math></p> $3^x = (3 \cdot 3)^2 \cdot 3 \cdot (3 \cdot 3 \cdot 3)^3$ $3^x = 3^2 \cdot 3^2 \cdot 3 \cdot 3^3 \cdot 3^3 \cdot 3^3$ $3^x = 3^{2+2+1+3+3+3}$ $\underline{x = 14}$ <p>(2)</p>	<p>B. <math>16 = 2^{3x-2}</math></p> $4^2 = 2^{3x-2}$ $(2 \cdot 2)^2 = 2^{3x-2}$ $2^{3x-2} = 2^2 \cdot 2^2$ $2^{3x-2} = 2^{2+2}$ $3x-2 = 4$ $\frac{3x}{3} = \frac{6}{3}$ $\underline{x = 2}$ <p>(2)</p>
<p>C. <math>p^5 \left(\frac{1}{p^2}\right) = p^x</math></p> $p^5 p^{-2} = p^x$ $p^x = p^{5+(-2)}$ $\underline{x = 3}$ <p>(2)</p>	

2. One circular ice skating stadium for children has a radius of  $x^2$  and the other for adults has a radius which is triple the first. Find the ratio of the area of the larger stadium to the area of the smaller stadium.

6 points.

Area of circle  $\pi r^2$

$$\text{Ratio} = \frac{\text{Area of the larger stadium}}{\text{Area of the smaller stadium}}$$

$$= \frac{\pi (3x^2)^2}{\pi (x^2)^2} = \frac{\pi 9x^4}{\pi x^4} = \frac{9}{1}$$

2 points

Sub total 8 points

The ratio of the area of the larger stadium to the area of the smaller is 9:1

3. Simplify or evaluate the following expressions. Write answers in simplest form.

<p>A. <math>10^{-2} \cdot 10^0</math>  <math>= \frac{1}{10^2} \cdot 1</math>  <math>= \frac{1}{100}</math></p>	<p>B. <math>[(-2)^3]^2</math>  <math>= [-2 \cdot 2 \cdot 2]^2</math>  <math>= [-8]^2</math>  <math>= 64</math></p>
<p>C. <math>(3x)^{-2} (-3x)</math>  <math>= 3^{-2} \cdot x^{-2} (-3x)</math>  <math>= \frac{1}{3^2 x^2} \cdot -3x</math>  <math>= \frac{1}{9 x^2} \cdot -\frac{3x}{1} = -\frac{1}{3x}</math></p>	<p>D. <math>(5x)^0 y^{-2}</math>  <math>= 1 \cdot \frac{1}{y^2}</math>  <math>= \frac{1}{y^2}</math></p>

8 points

4. Make a table of values for the exponential function  $y = \left(\frac{1}{3}\right)^x$
- Show how you evaluated at least one input output pair in your table.

x	-2	-1	0	1	2
$y = \left(\frac{1}{3}\right)^x$	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

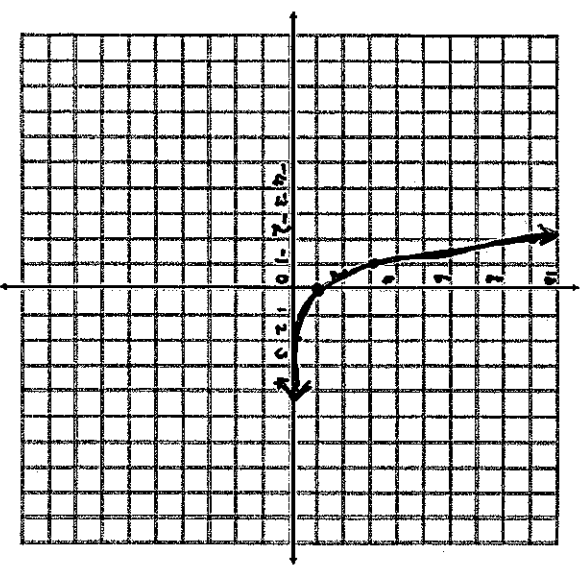
$$y = \left(\frac{1}{3}\right)^x \quad \text{when } x = -2$$

$$= \left(\frac{1}{3}\right)^{-2}$$

$$= \frac{1}{12} \div \frac{1}{3^2}$$

$$= 1 \times 3^2 = 1 \times 9 = 9.$$

- Use your table to graph this function.



3 points

**Sub total 11 points**

5. Simplify the following expressions. Use only positive exponents in your answer.

<p>A.</p> $\frac{5x^2y \cdot 6x^4y^2}{3xy^2 \cdot x^2y^2}$ $= \frac{30x^{2+4} \cdot y^{1+2}}{3x^{1+2} \cdot y^{2+2}}$ $= 10x^{6-3} \cdot y^{3-4}$ $= 10x^3 \cdot \frac{1}{y}$ $= \frac{10x^3}{y}$	<p>B.</p> $x^{-8} \cdot x^{10} \left( \frac{y^3}{y^5} \right)^{-2}$ $= \frac{1}{x^8} \cdot x^{10} \left( \frac{y^{-6}}{y^{-10}} \right)$ $= x^{10-8} \cdot \frac{1}{y^6} \cdot \frac{1}{y^{-10}}$ $= x^2 \cdot \frac{1}{y^6} \cdot y^{10}$ $= x^2 \cdot y^{10-6}$ $= x^2y^4$
---	--

4 points

6. The human body has  $1 \times 10^{12}$  cells. There are  $3 \times 10^{10}$  red blood cells. Find the ratio of red blood cells to the total number of cells and write the number in scientific notation.

$$\text{Ratio} = \frac{\text{Red blood cells}}{\text{Total number of cells}}$$

$$= \frac{3 \times 10^{10}}{1 \times 10^{12}}$$

$$= 3 \times 10^{-2}$$

2 points

**Sub total 6 points**

7. Write the number in decimal form.

<p>A. <math>0.759 \times 10^6</math>  <math>= 0.7590000</math>  <math>= \underline{759\ 000}</math></p>	<p>B. <math>52.4 \times 10^{-4}</math>  <math>= 0.00524</math>  <math>= \underline{0.000524}</math></p>
---	---

4 points

8. A population of 40 pheasants is released in a wild life preserve. The population doubles each year. What is the population after 4 years?

- Write an exponential growth model
- Evaluate the pheasant population after 4 years?
- Graph the population growth over four years.

$t$  represents time in years -  $y$  represents pheasants

$$y = 40(1+1)^t$$

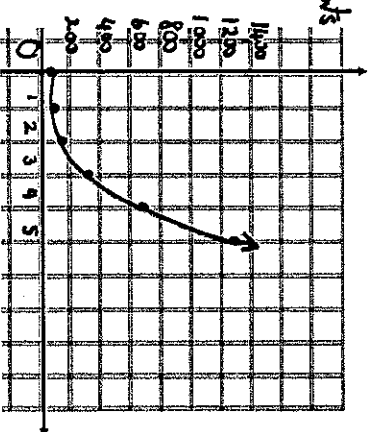
$$= 40 \cdot 2^4$$

$$= \underline{640}$$

The population after 4 years is 640 pheasants

$t$	0	1	2	3	4	5
$y$	40	80	160	320	640	1280

3 points



9. Write an exponential growth model for the profit.

A business has a \$ 5000 profit in 1990. Then this profit increased by 15% per year for the next 10 years.

$$y = 5000(1 + 0.15)^t$$

$$= \underline{5000(1.15)^{10}}$$

2 points

**Sub total 9 points**

10. Evaluate or simplify the following expressions without using a calculator

<p>A.</p> $\sqrt{432}$ $= 2 \cdot 2 \cdot 3 \cdot \sqrt{3}$ $= \underline{12\sqrt{3}}$	<p>Working</p> $\begin{array}{r} \cancel{2} \quad \cancel{2} \quad \cancel{3} \quad \textcircled{3} \\ \hline 432 \\ \hline 108 \\ \hline 54 \\ \hline 27 \\ \hline 9 \\ \hline 3 \\ \hline 1 \end{array}$	<p>B.</p> $\sqrt{0.0025x^4y^6z^5}$ $= \underline{0.05x^2y^3z^2\sqrt{z}}$	<p>Working</p> $\begin{array}{c} \cancel{xx} \quad \cancel{xx} \\ \sqrt{y \cdot y \cdot y \cdot y \cdot y} \\ \cancel{zz} \quad \cancel{zz} \quad z \end{array}$
<p>C.</p> $(-2\sqrt{7})^2$ $= (-2\sqrt{7})(-2\sqrt{7})$ $= 4\sqrt{7 \cdot 7}$ $= 4\sqrt{49}$ $= 4 \cdot 7$ $= \underline{28}$		<p>D.</p> $\frac{12}{8\sqrt{15}}$ $= \frac{4}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}}$ $= \frac{4\sqrt{15}}{15}$	

**Sub total 8 points**

13. Solve the equations. Write the solution(s) as simplified as possible.

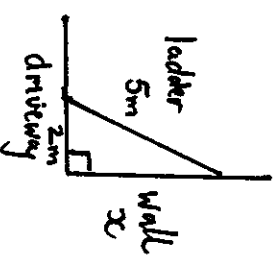
A.	B.
$\frac{3a^2}{3} = \frac{147}{3}$ $\sqrt{a^2} = \sqrt{49}$ $a = \pm 7$	$6x^2 - 54 = 0$ $6x^2 - 54 + 54 = 0 + 54$ $\frac{6x^2}{6} = \frac{54}{6}$ $\sqrt{x^2} = \sqrt{9}$ $x = 3$

4 points

14. A ladder is 5m long. Its foot is on a flat driveway 2m from the base of a vertical wall. How far up the wall will the top of the ladder reach?

- Draw a sketch of the ladder leaning against the wall
- Using the 4 step problem solving process find out how far up the wall the ladder will reach
- Give your answer in its simplest form.

Let  $x$  be the height of the wall at the point where the top of the ladder touches it.



$$a^2 + b^2 = c^2$$

$$2^2 + x^2 = 5^2$$

$$x^2 = 5^2 - 2^2$$

$$x^2 = 25 - 4$$

$$\sqrt{x^2} = \sqrt{21}$$

$$x = \sqrt{21}$$

4 points

The ladder touches the wall  $\sqrt{21}$  meters from the ground.

Sub total 8 points