Name: _____

Date: _____

Practice Test - BLACK

Score: $\frac{}{23}$

Factors, Fractions, and Exponents

Clearly SHOW or EXPLAIN how you arrive at ALL your answers !!!

1. Divisible by a Dozen

Take any three consecutive integers. Multiply the first times the square of the second, then multiply by the third.

Example:
    If you use 2, 3, 4 then you would have 2 * 3^2 * 4, which equals 72.

A. Try this with four different groups of three consecutive integers. Is each result divisible by 12?   (1 point)

B. Show or explain why this is always true no matter what three integers you start with.
    (1 point)

2. For what value of n is the five-digit number 7n,933 divisible by 33? To do this problem you need to know the divisibility rule for 11. The **divisibility rule for 11** is: A number is divisible by 11 if the difference between the sum of the odd numbered digits (1st, 3rd, 5th...) and the sum of the even numbered digits (2nd, 4th...) is 0 or is divisible by 11

    (1 point)

3. Find a way of writing 1,000,000 as a product of two numbers, neither of which ends in zero.   (1 point)

4. List all numbers less than 100 with exactly 5 factors. Find the next two numbers, each greater than 100, that have exactly five factors.   (1 point)

5. What is the least positive integer that has each of the first eight positive integers as factors?   (1 point)

6. If $x \oplus y = (x^y)^x$, what is the units digit of $7 \oplus 5$?   (1 point)

7. The five-digit whole number 3a,7b1 is a perfect square. What is the greatest possible value for the product ab?   (1 point)

8. What is x, if $x^{12} = 2$? Express your answer as a decimal to the nearest hundredth.
(1 point)

9. John's Cousins:

   "This might interest you, professor," said John. "My age and the ages of each of my three distant cousins are all prime numbers, and the sum of our ages is 50.

   "In that case," said the professor, who knew John's age, "I can tell you the ages of your three cousins."

   You do not share the professor's advantage of knowing John's age to start with, but nevertheless, can you tell the ages of his cousins? (note that the number 1 is not considered to be a prime). (1 point)

10. A digital, 12 hour clock shows hours and minutes. During what fraction of the day will the clock show the digit 1 in its display? Express your answer as a common fraction. (1 point)

11. Jill gives half her cards to Bill. Bill then gives half of his cards to Jill. Jill and Bill repeat this two-step process twice more. If Jill now has 37 cards and Bill has 19 cards, with how many cards did Bill start? (1 point)

12. Express the reciprocal of 3.8 as a common fraction. (1 point)

13. Solve for x. Provide work or an explanation that makes the thinking that leads to your answer clear.   (2 points)
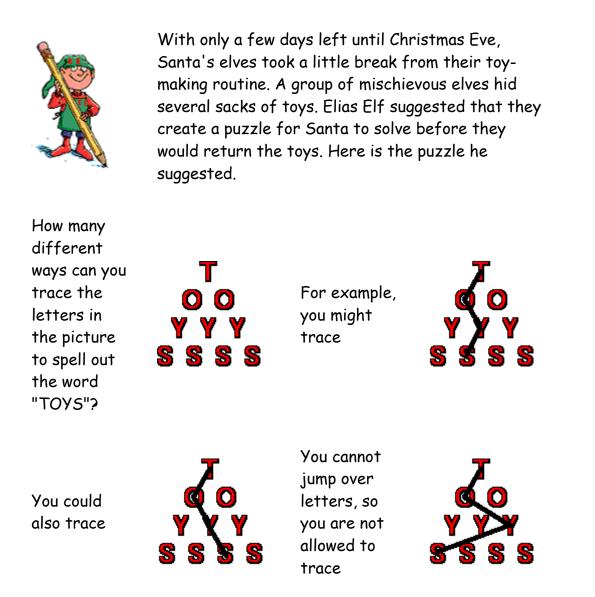
A. $30^{17} = 6^x 5^x$

B. $3^3 \bullet 9^3 \bullet 27^3 \bullet 81^3 \bullet \bullet \bullet \bullet \bullet 2187^3 = 3^x$

14. Evaluate $(7^3 + 7^3 + 7^3 + 7^3 + 7^3 + 7^3 + 7^3)^{\frac{1}{2}} =$ (1 point)

15. One trillion is 10^n divided by one-millionth. What is the value of n?   (1 point)

16. Ocean Water Molecules:

A. According to the World Book Encyclopedia, there are about 326 million cubic miles of water on the earth. One cubic mile is 5280^3 cubic feet. About how many cubic feet of water are there on the earth? Express your answer in scientific notation.   (1 point)

B. A thimble the size of your thumb holds about 5×10^{-4} cubic foot of water. How many thimblefuls of water are there on the earth?   (1 point)

C. A cubic foot of water has about 9.47×10^{26} water molecules. Which is greater, the number of water molecules in a thimbleful of water or the number of thimblefuls of water on the earth?   (1 point)

D. How many times as great?   (1 point)

With only a few days left until Christmas Eve, Santa's elves took a little break from their toy-making routine. A group of mischievous elves hid several sacks of toys. Elias Elf suggested that they create a puzzle for Santa to solve before they would return the toys. Here is the puzzle he suggested.

How many different ways can you trace the letters in the picture to spell out the word "TOYS"?

```
    T
  O   O
 Y  Y  Y
S  S  S  S
```

For example, you might trace

```
    T
  O   O
 Y  Y  Y
S  S  S  S
```

You could also trace

```
    T
  O   O
 Y  Y  Y
S  S  S  S
```

You cannot jump over letters, so you are not allowed to trace

```
    T
  O   O
 Y  Y  Y
S  S  S  S
```

Bobby Elf thought that this was a good idea, but he thought the word was too easy. Emilee Elf suggested that they use a bigger word to make it more difficult. Susann Elf thought that the word "GIFTS" would be a better choice. Then Maggie Elf suggested the word "PRESENTS."

```
              P
            R   R
          E   E   E
        S   S   S   S
      E   E   E   E   E
    N   N   N   N   N   N
  T   T   T   T   T   T   T
S   S   S   S   S   S   S   S
```

Use your mathematical skills to discover a pattern that will help Santa to solve the puzzle no matter what word the clever elves finally decide to use.

Clearly state a general rule that will tell Santa the total number of ways to trace the letters in any word. Show how you would apply your rule to the three words (toys, gifts, presents) suggested by the elves.   (1 point)

18. Locker Dilemma: You are about to enter your brand new school for the first time. The teachers, however, have gotten together and decided to perform a little ritual. All 150 students of the school need to line up and enter the school one at a time. The first student entering will open all 150 lockers. The second student will enter and close every second locker. The third student will change every third locker… and so on. You are the last in line! But as you are waiting for your turn you realize you can figure out which lockers will be open after your turn. You amaze your teachers! Which lockers are open?   (1 point)