Blue Practice Test Solutions – Unit 8 – Area and Volume

1. Many times students are not sure when they can substitute in a concrete example for an abstract situation. Since this problem does not suggest that this answer is specific to a particular rectangle, the easiest path to the solution may be to come up with a rectangle that we think would be easy to work with. Notice we are increasing the length by 25%, so using a measurement for the length that is a multiple of 4 might be best. Let's go ahead and start with a length of 4 units. Since we're not sure what's going to happen to the width, but it's going to decrease by a percent, let's make the width 100 units since it will be easy to determine a percent increase or decrease. So now we have a 4 by 100 rectangle that has an area of 400 square units. We are told that the length increases by 25%. This means our length is going to grow from 4 units to 5 units. We desire to keep the same area of 400 square units, and we're looking for our new width, w. Consider the equation $5w = 400$. By dividing both sides of the equation by 5, we see that $w = 80$. Notice our width went from 100 to 80. That's a decrease of 20 units, and since it's 20 units out of 100 units, it's a 20% decrease.

   This problem can be solved without a concrete example. Let the original rectangle have length = L and width = W. The area of the original rectangle is then $A = L \times W$. If we increase the length by 25%, it becomes $1.25L$. Now we need to find a percent of the width to fill into this equation to make it work: $(1.25L) \times (?\,W) = L \times W$, since our goal is to keep the same area. Notice we need the 1.25 to multiply with this missing percent to give us a product of 1. Then we'll just be left with $L \times W$ on the left side of the equation as well. Since we'd like the 1.25 and the missing percent to multiply to 1, we're really looking for the reciprocal of 1.25. Remember that $1.25 = \frac{5}{4}$ in its fraction form. The reciprocal of this is $\frac{4}{5}$ which is 80%. Notice that $(\frac{5}{4}L) \times (\frac{4}{5}W) = L \times W$.

2. . The area of rectangle $ABCD$ is $6 \times 4 = 24$ square units. Triangles AEH and CFG together make a rectangle that is $3 \times 4 = 12$ square units. Likewise, triangles DGH and BEF together make a rectangle that is $1 \times 2 = 2$ square units. Subtracting 12 and 2 from 24, we find that parallelogram $EFGH$ has an area of 10 square units. The ratio of the area of parallelogram $EFGH$ to rectangle $ABCD$ is $\frac{10}{24} = \frac{5}{12}$.

3. Each of the pepperoni slices has area $\pi(\frac{1}{2})^2 = \frac{\pi}{4}$ square inches. The pizza has area $\pi(7)^2 = 49\pi$ square inches. The difference, then, is $49\pi - 28(\frac{\pi}{4}) = \pi(49 - 7) = 42\pi$ square inches.

4. 427.9 cm2

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4. Area of shaded part of larger circle
= Area of larger circle − $76.5\ cm^2$

Area of shaded part of larger circle = $\pi r^2 - 76.5\ cm^2$
$= 3.14 \times 11cm \times 11cm - 76.5cm^2$
$= 379.94\ cm^2 - 76.5\ cm^2$
$= \boxed{303.44\ cm^2}$

Area of shaded part of smaller circle
= Area of smaller circle − $76.5\ cm^2$
$= \pi r^2 - 76.5\ cm^2$
$= 3.14 \times 8cm \times 8cm - 76.5\ cm^2$
$= 200.96\ cm^2 - 76.5\ cm^2$
$= \boxed{124.46\ cm^2}$

Total area of the shaded parts
$= 303.44\ cm^2 + 124.46\ cm^2$
$= \boxed{427.9\ cm^2}$

5. 69 cm

5. The arcs in the figure are each quarter circles of radius $14+7 = \underline{21 \text{ cm}}$ and radius $\underline{14 \text{ cm}}$ respectively

Circumference of larger full circle $= 2\pi r$

$= 2 \times \frac{22}{\cancel{7}_1} \times \cancel{21}^3 \text{ cm}$

$= 132 \text{ cm}$

Length of larger arc $= \frac{132 \text{ cm}}{4} = 33 \text{ cm}$

Circumference of smaller full circle $= 2\pi r$

$= 2 \times \frac{22}{\cancel{7}_1} \times \cancel{14}^2 \text{ cm}$

$= 88 \text{ cm}$

Length of smaller arc $= \frac{88}{4} \text{ cm} = 22 \text{ cm}$

Perimeter of figure shown $=$

$= 33 \text{ cm} + 22 \text{ cm} + 7 \text{ cm} + 7 \text{ cm}$

$= \boxed{69 \text{ cm.}}$

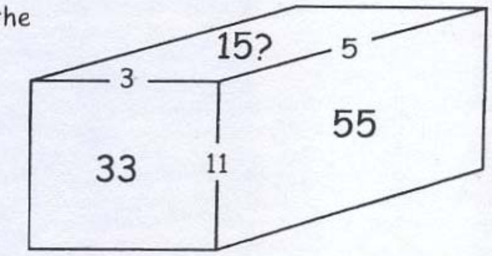
6. Doubling the radius leads to a volume of $\frac{4\pi r^2 h}{3}$. Tripling the height yields $\pi r^2 h$. Thus doubling the radius has a greater impact on the volume.

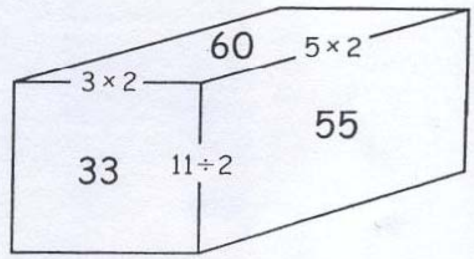
7. The volume of the box is now $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ of its former volume, so it has decreased by $\frac{7}{8}$ or **87.5%.**

8. A cube with an edge length of 30 cm has a volume of $30^3 = 27{,}000$ cubic centimeters. The sphere inside the box has a radius of $30 \div 2 = 15$ cm. And a volume of $v = 4/3 \times \pi \times r^3 = 4/3 \times \pi \times 15^3 = 4/3 \times \pi \times 3375 = 4500\pi$. The space in the box not occupied by the sphere is **$27{,}000 - 4500\pi$.**

9. If we take this solid and assign variables to the three dimensions (x, y and z), then in some order $xy = 33$, $yz = 55$ and $xz = 60$. This is because the product of each pair of these dimensions will give us the area of one of the faces. Notice what we can now do with these three equations: $xy \times yz \times xz = 33 \times 55 \times 60$. By simplifying and factoring some numbers a bit, we have $x^2y^2z^2 = 3 \times 11 \times 5 \times 11 \times 2 \times 2 \times 3 \times 5 = 2^2 \times 3^2 \times 5^2 \times 11^2$. Taking the square root of both sides of this new equation yields an equation for the volume: $xyz = 2 \times 3 \times 5 \times 11 = 330$ cubic units.

Let's also take a look at a different representation of the problem using the original picture. Since the areas of the faces must come from the product of the dimensions, it makes sense to try to find some common factors of the face areas and see if those work as the dimensions of the box. Notice first that 33 and 55 both have a common factor of 11. Let's then assume that the edge these two faces share is 11 inches long. From there, we can figure what the other dimensions of the box would need to be to create the areas of 33 and 55. Notice, though, that when we put in the other dimensions of 3 and 5, that creates a face with area 15, and we need the area to be four times that big. What if we increased the 3 and 5 edges each by a factor of 2, which would increase the area of their shared face by the desired factor of 4? Then we have to adjust the edge that is currently 11 inches by decreasing it by a factor of 2, in order to keep the area of each of the other two faces the same. We'll see this gives us the face areas we need, and our edge lengths are 6, 10 and $\frac{11}{2}$, which give us a volume of $6 \times 10 \times \frac{11}{2} = 330$ cubic units.





10. Each face of the large cube still has all of its $21 \times 21 = 441$ square centimeters of area, but the 9 square centimeters in the centers are now at the bottom of the hole and there are $3 \times 9 \times 4 = 108$ additional square centimeters on the four walls of each hole. That's $441 + 108 = 549$ square centimeters on each of 6 faces of the cube, for $549 \times 6 =$ **3294 square centimeters** total.

11.

Length of larger arc $= \frac{132\text{ cm}}{4} = 33$ cm

Circumference of smaller full circle $= 2\pi r$

$= 2 \times \frac{22}{\cancel{7}_1} \times \cancel{14}^2$ cm

$= 88$ cm

Length of smaller arc $= \frac{88}{4}$ cm $= 22$ cm

Perimeter of figure shown $=$

$= 33\text{ cm} + 22\text{ cm} + 7\text{ cm} + 7\text{cm}$

$= \boxed{69\text{ cm.}}$

12. (a). The cross-section is a circle.

Area of the circle $= \pi r^2$

$= 3.14 \times 2\text{m} \times 2\text{m}$

$= 12.56\ \text{m}^2$

$\doteqdot 13\ \text{m}^2$

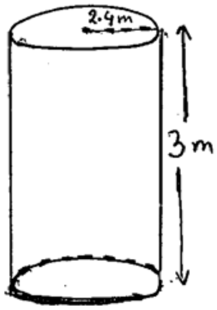
(b) The cross-section is a rectangle.

Area of the rectangle $= l \times w$.

$= 12\text{ m} \times 4\text{ m} = 48\text{ m}^2$

[Note: $w = 2 \times \text{diameter} = 2 \times 2\text{m} = 4\text{m}$]

13.



3

To find the outside area of the cylinder that the sealing is applied to, we need to find the Surface Area of the cylinder

$$S.A = 2\pi rh + 2\pi r^2$$
$$= 2\cdot(3.14)\cdot(2.4m)\times 3m + 2\cdot(3.14)\cdot(2.4m)(2.4m)$$
$$= 45.216\ m^2 + 36.173\ m^2$$
$$= 81.389\ m^2$$
$$\doteq 81.4\ m^2$$

Cost of sealent = \$8.50/m²

The cost of coating the outside area of the cylinder = $81.4 \times \$8.5 = \boxed{\$691.9}$

$\doteq \boxed{\$692}$

The surface area inside the cylinder is the same as the surface area outside the cylinder

So, the cost of coating one of these tanks on the inside and the outside = $\$691.9 \times 2$

$= \boxed{\$1383.8}$

$\doteq \boxed{\$1384}$

14. The stake should be placed in the middle of the square yard.

The rope should be 75 feet long, and the goat will be able to eat grass in an area of 17,662.5 square feet.

4,837.5 square feet will still have to be mowed.

It will take about an hour.

FYI:

Area of yard: 22,500 square feet;	3,240,000 square inches
Area of circle: 17,662.5 square feet;	2, 543,400 square inches
Area of leftover: 4,837.5 square feet;	696,600 square inches

The exact time needed to cut the grass is 1.16 hours.